B.SC - III SEMESTER

MATHEMATICS : REAL ANALYSIS

Important Questions

Unit - I :

1. limit of a sequence exists then it is unique?
2. prove That The sequence $\left\{\frac{(-1)^{n+1}}{n}\right\}$ is converges to '0'?
3. If $s_n = \sqrt{n+1} - \sqrt{n}$ prove That $\lim s_n = 0$
4. Show That $\lim \sqrt{\frac{n+1}{n}} = 1$
5. prove That $\lim \left(\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2}\right) = 0$.
6. Discuss The nature of The sequence $\{r^n\}$ $\forall$ $-1 < r \leq 1$
7. If $s_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}$, prove That $\{s_n\}$ is Convergent?
8. prove That $\lim_{n\to\infty} \left\{\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right\} = 1$
9. A monotonic sequence is bounded iff it is Convergent?
10. prove That The sequence $s_n = \left(1+\frac{1}{n}\right)^n$ is Convergent?
11. prove That The sequence $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ is Convergent?

(12) Every Cauchy sequence is Convergent?

(13) prove That $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ is not convergent?

(14) Define The following terms

(a) Sequence (b) Convergent sequence (c) Bounded sequence
(d) Cauchy sequence (e) oscillating finite sequence

(15) prove That $S_n = 2 - \frac{1}{2^{n-1}}$ is Convergent?

UNIT-II

(1) State and prove Geometric Series.

(2) State and prove Auxiliary (or) Harmonic Series.

(3) Test The convergence of the following series.

(a) $\sum_{n=1}^{\infty} \frac{1}{2^n+3^n}$ (b) $\sum_{n=1}^{\infty} \sqrt{n+1}-\sqrt{n}$ (c) $\sum_{n=1}^{\infty} \sqrt{n^2+1}-n$

(d) $\sum_{n=1}^{\infty} \left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$ (e) $\sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)(n+2)}$

(4) State and prove Cauchy's n^{th} root test.

(5) Test The Convergence of The following Series

(a) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{-n}$ (b) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$ (c) $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+5}\right)^n$

(6) State and prove Ratio Test (D. Alembert's)

(7) Test The Convergence of $\sum_{n=1}^{\infty} \frac{n^4}{n!}$

(8) Test the Convergence of the following series

(a) $\sum_{n=1}^{\infty} \frac{2^n-2}{2^n+1} x^n \ (x>0)$ (b) $\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^3}$

(c) $\sum_{n=1}^{\infty} \frac{x^n}{x^n+a^n}$ (d) $\sum_{n=1}^{\infty} \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n} x^{n-1} \ (x>0)$

(9) State and prove Cauchy's Condensation Test,

(10) The Auxiliary Series $\sum_{n=1}^{\infty} \frac{1}{n(\log n)^p}$ Converges if $p>1$ and diverges if $p \leq 1$

(11) State and prove Leibnitz Test.

(12) Examine the Convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n}\right)}{n}$

(13) prove That $\sum (-1)^{n-1} \cdot \frac{x^n}{n}$ is Convergent for $-1<x\leq 1$

UNIT-III

SEQUENCES & SERIES OF FUNCTIONS

(1) ~~Test the P~~

Determine The Radius of Convergence and Exact interval of Convergence of The power series

(a) $\sum_{n=1}^{\infty}\left(\frac{n^3}{3^n}\right)x^n$ (b) $\sum\left(\frac{2^n}{n!}\right)x^n$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

(d) $\sum \frac{(-1)^n}{n^2 4^n} x^n$ (e) $\sum \frac{1}{(n+1)^2 2^n} x^n$

(2) prove that if $f_n \to f$ converges uniformly a set 'S' and if $g_n \to g$ converges uniformly on 'S' then prove that $f_n \pm g_n \to f \pm g$ converges uniformly on S.

(3) Show that $\sum_{n=1}^{\infty} \frac{x^n}{n^2 2^n}$ has radius of convergence '2' and the series converges uniformly to a continuous function on $[-2,2]$

(4) Show that $\sum \frac{x^n}{1+x^n}$ converges uniformly on $[0,a]$ for each $0<a\leq 1$

(5) prove that $\sum_{n=1}^{\infty} x^{n-1}(1-x)$ uniformly converges on $[-\frac{1}{2},\frac{1}{2}]$

(6) If $f_n(x)=\frac{nx}{1+n^2x^2}$, $\forall x\in[0,1]$ then prove that $\{f_n\}$ is not uniformly convergent on $[0,1]$

(7) Let $f_n(x)=\frac{n+\cos x}{2n+\sin^2 x}$ $\forall x\in R$. Show that $\{f_n(x)\}$ converges uniformly on 'R' and also evaluate

$$\lim_{n\to\infty}\int_2^7 f_n(x)\,dx$$

(8) Show that $f_n(x)=\frac{x}{1+nx^2}$ $\forall x\in R$ is uniformly converges.

UNIT- IV (3)

Reimann Integration

(1) Define the terms

(a) partition (b) Norm (c) lower & upper Reimann Sum

(d) lower & upper Reimann Integral.

(2) If $f(x) = x$ on $[0,1]$ and $P = \{0, \frac{1}{3}, \frac{2}{3}, 1\}$ then find $L(P,f)$ & $U(P,f)$

(3) Every Constant function is Reimann Integrable on $[a,b]$

(4) If $f \in R[a,b]$ and m, M be the inf & sup of f on $[a,b]$ then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

(5) State & prove Necessary & Sufficient Condition for Reimann Integrability.

(6) State & prove fundamental theorem of Integral calculus.

(7) If $f: [a,b] \to R$ is monotonic on $[a,b]$ then prove that f is Reimann Integrable on $[a,b]$.

(8) Show that $\left| \int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)\,dx \right| \le \frac{16\pi^3}{3}$

(9) Let f and g be Reimann Integrable function on $[a,b]$ then prove that $f+g$ is Reimann Integrable and

$$\int_a^b (f+g)(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx.$$

(10) If f is bounded on $[a,b]$ then

$$\underline{\int_a^b} f(x)\,dx \le \overline{\int_a^b} f(x)\,dx.$$

(11) Show that $f(x)=3x+1$ is Integrable on $[1,2]$ and $\int_1^2 (3x+1)\,dx = 11/2$

(12) prove that $f(x)=x^2$ is Integrable on $[0,a]$ and

$$\int_0^a x^2\,dx = \frac{a^3}{3}$$

(13) If $f:[a,b]\to\mathbb{R}$ is defined by $f(x)=x$ then prove

$$\int_a^b f(x)\,dx = \frac{1}{2}(b^2-a^2).$$

(14) P.T $f(x)=\sin x$ is Integrable on $[0,\pi/2]$ and

$$\int_0^{\pi/2} \sin x\,dx = 1$$