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MATHEMATICS

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SEMESTER - IV

UNIT-I & II

(1) Define the terms
(a) Group (2) Normal subgroup (3) permutation group.
(4) cyclic group. (5) Center of a group. (6) order of an element
(7) Index of a subgroup.

(2) Show that the set $G=\{x \mid x=2^a 3^b \text{ \& } a,b\in Z\}$ is a Group under multiplication.

(3) prove that the set $G=\left\{A_\alpha=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix} \middle| \alpha\in R\right\}$ forms a group w.r.to matrix multiplication.

(4) If G is a group then p.T $(ab)^{-1}=b^{-1}.a^{-1}$ $\forall$ $a,b\in G$

(5) If G is a group and 'a' is an element of a group G
[Then $\exists$ $a^n=e$ such that $a^m=e \Leftrightarrow n|m$.

(6) prove that union of two subgroups is also a subgroup of G $\Leftrightarrow$ one is contained in another.

(7) prove that the intersection two subgroups is also a subgroup of G.

(8) If H is a normal subgroup of a group $G \Leftrightarrow xHx^{-1}=H$.

(9) If 'G' is a group. Then 'H' is a subgroup of index '2'. Then show that 'H' is ~~also~~ ~~sub~~ normal subgroup of 'G'.

(10) State and prove Cancellation laws

(11) State and prove Cayley's Theorem on permutation of groups.

(12) State and prove Lagrange's Theorem on finite groups.

(13) prove That n^{th} roots of unity forms a cyclic group of order n.

(14) prove that The group $G = \{1, 3, 5, 7\}$ forms an abelian group w.r.to multiplication modulo 8.

(15) Let 'a' be an element of order 'n' in a group 'G' and let 'k' be positive Integer Then show That

(1) $\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$ (2) $|a^k| = \dfrac{n}{\gcd(n,k)}$

(16) Define The terms (1) Homomorphic Image

(2) Kernel of Homomorphism

(3) Isomorphism

(4) automorphism.

(17) Let G and $\bar{G}$ be two groups Then 'f' is homomorphism and onto from G to $\bar{G}$ with $\ker f$ is isomorphism $\Leftrightarrow \ker f = \{e\}$.

(18) State and prove fundamental theorem of homomorphism

(19) If $f = (1\,2\,3\,4\,5\,8\,7\,6)$ & $g = (4\,1\,5\,6\,7\,3\,2\,8)$ are the cyclic permutations Then p.T $(fg)^{-1} = g^{-1}f^{-1}$.

(20) (1) Every prime order group is cyclic

(2) The order of a cyclic group is equal to ~~its~~ order of its generator.

(21) list of all cyclic subgroups of $U(30)$. Is $U(30)$ is cyclic?

22. Consider the set $\{4, 8, 12, 16\}$. Show that this set is a group under multiplication 20 using Cayley Table. Is it cyclic. If so find its generators.

23. Find all the cosets of $H = \{1, 15\}$ in the group $G = U(32)$.

24. State and prove Fermat's little Theorem and Compute (1) $5^{15} \bmod 7$ (2) $7^{13} \bmod 11$

25. (1) Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \middle| a, b, d \in R,\ ad \neq 0 \right\}$. Is H is a normal Subgroup of $GL(2, R)$.

(2) Let $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \middle| a, d \in R \right\}$, then prove that H is a normal Subgroup of $GL(2, R)$

26. Let ϕ be a Homomorphism from a group G to a group $\bar{G}$ Then 1) $\ker\phi$ is a Subgroup of G.
2) $\phi(G)$ is a Subgroup of $\bar{G}$.

27. (1) Let 'ϕ' be a homomorphism from a group G to a group $\bar{G}$ Then prove that $\ker\phi$ is a normal Subgroup of G.
(2) Let $\phi: Z \to R^+$ is defined by $\phi(a) = 2^a$ then prove that (1) ϕ is homomorphism (2) Homomorphic Image (3) Kernel.

28. 1) Find the number of generators of Z_{84}, Z_{12}, Z_8.

29. 2) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ are two permutations in S_6 Then Compute $\sigma\tau^2$ and $\sigma\tau\sigma^{-1}$.

30. 1) Define cosets. Find all cosets of the Subgroup $4Z$ of Z
(2) Every Quotient group of an abelian group is abelian.

UNIT-III Rings & Fields

(1) define The terms
(1) Ring (2) Integral domain (3) field
(4) division Ring (5) characteristic of a Ring
(6) zero divisors of a Ring (7) Idempotent elements of a Ring
(8) Nilpotent elements (9) Units of a Ring.

(2) Define Boolean Ring. If R is Boolean Ring Then P.T
1) $a+a=0 \ \forall a \in R$ (2) $a+b=0 \Rightarrow a=b$ (3) Every Boolean Ring is abelian.

(3) Let $*$ and 'o' be defined on Z i.e $a*b=a+b-1$ and $aob=a+b-ab$ Then show that $(Z,*,o)$ is a ring.

(4) Define Subring. prove That the intersection of two subrings is a subring of 'R'.

(5) If 'R' is a Ring the set of matrices $S=\left\{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} / a,b,c \in Z\right\}$ is a Subring of R.

(6) prove that $2Z \cup 3Z$ is not a Subring of 'Z'.

(7) Show That Every Field is an Integral domain.

(8) prove That Every finite Integral domain is field.

(9) Let $R=\{0,2,4,6,8\}$ Then prove That 'R' is a field under addition & multiplication modulo 10.

(10) 1) Find all The Solutions of $x^2+x-6=0$ in Z_{14}
2) Find all the solutions of $f(x)=x^2-5x+6=0$ in Z_8, Z_{12}, Z_7.

(11) Prove that the only Idempotent elements of an Integral domain are 0 and 1 (3)

(12) (1) Find all the zero divisors, idempotent, nilpotent and units of the ring (1) Z_{10} (2) Z_{11} (3) Z_{12}

(2) Find all units, zero divisors, idempotent elements and nilpotent elements in $Z_3 \oplus Z_6$.

(13) Show that intersection of two ideals is an ideal of R.

(14) If U_1 & U_2 are two ideals of 'R' then show that $U_1 \cap U_2$ is an Ideal of $R \iff U_1 \subset U_2$ (or) $U_2 \subset U_1$

(15) (1) If 'R' is a Ring then show that $S = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \middle| a, b \in Z \right\}$ is Right ideal but not left ideal.

(2) If 'R' is a Ring then show that $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \middle| a, b \in Z \right\}$ is left ideal but not Right ideal.

(16) define the terms (1) principle ideal
(2) maximal ideal
(3) prime Ideal.

(17) Find all prime and maximal ideals of Z_6

(18) If 'R' is a commutative Ring with unity and $\frac{R}{M}$ is a field $\iff$ M is maximal ideal of 'R'.

(19) 1) prove that the characteristic of Boolean Ring is '2'.

2) let x, y be the elements of a commutative ring 'R' of characteristic '2'. Show that $(x+y)^2 = x^2+y^2 = (x-y)^2$

(20) (1) Show that the set of Real numbers $a+b\sqrt{2}$ with a & b are real numbers is a field.

(2) prove that the ring of Gaussian integers $Z[i] = \{a+bi \mid a,b \in Z\}$ is an Integral domain.

UNIT-IV

RING OF HOMOMORPHISMS & POLYNOMIAL RINGS

(1) State and prove fundamental theorem of Ring Homomorphisms (or) First Isomorphism theorem

(2) (1) Determine all Ring Homomorphism from the ring Z_{20} to Z_{30}

(2) " " " " $Z_6 \to Z_6$

(3) If A is an Ideal of R then the Quotient Ring $\frac{R}{A}$ is homomorphic image of R.

(4) If $\phi: Z_4 \to Z_{10}$ is defined by $\phi(x) = 5x \; \forall \; x \in Z_4$ then P.T ϕ is a homomorphism.

(5) Consider $f(x) = 2x^3+x^2+2x+2$ and $g(x) = 2x^2+2x+1$ in $Z_3(x)$ then find $f(x)+g(x)$ and $f(x)g(x)$

(6) State and prove division algorithm in polynomial rings.

(7) State and prove factor theorem

(8) ~~Let~~ If $f(x) = 7+9x+5x^2+11x^3-2x^4$ & $g(x) = 3-2x+7x^2+8x^3$ are polynomial in $Z_7(x)$ then prove that

(1) $\deg(f(x)+g(x)) = 4$ (2) $\deg(f(x) \cdot g(x)) = 7$

(9) Find all zeros in Z_5 of $2x+x^2+x^3+x^5$ in $Z_5(x)$

(10) If $f(x) = x^4-3x^3+2x^2+4x-1$ and $g(x) = x^2-2x+3$ are polynomials in $Z_5(x)$. then find $q(x)$ and $r(x)$ of the division algorithm.