DEPARTMENT OF MATHEMATICS

Question Bank - Paper - III

SVDC (MBNR)

UNIT - I

VECTOR SPACES

(1) (a) Define vector space & vector subspaces.

(b) Let V(F) be a vector space. A non empty set $W \subseteq V$. the necessary and sufficient condition for W to be a subspace of V is $a, b \in F$ and $\alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$

(2) The intersection of any two subspaces W_1 and W_2 of vectorspace V(F) is also subspace.

(3) The union of two subspaces is a subspace $\Leftrightarrow$ one is contained in the other

(4) The linear span L(S) of any subset S of a vector space V(F) is a subspace of V(F).

(5) Define LI & LD of vectors.

(6) Every superset of a LD set is LD.

(7) Every non-empty subset of a LI set of vector is LI.

(8) Determine $\{(-1,2,1)\ (3,0,-1)\ (-5,4,3)\}$ is LI or LD (Practice IIly problems)

[If all scalars are zeros LI & any one non zero is LD]

(9) Let V(F) be a vector space and $S = \{\alpha_1, \alpha_2 \cdots \alpha_n\}$ is a finite subset of non-zero vectors of V(F). then S is LD $\Leftrightarrow$ some vector $\alpha_k \in S$, $2 \le k \le n$ can be expressed as a linear combination of its preceding vectors.

(10) Define Basis and state & Prove Basis extension thm.

(11) show that the set $s = \{(1,2,1)\ (2,1,0)\ (1,-1,2)\}$ forms a basis of $V_3(F)$ [Prove LI and $L(S) = V_3$] (Practice the Problem)

(12) Let W_1 and W_2 be two subspaces of a finite dimensional vector space $V(F)$. Then $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

(13) Let W be a subspace of a finite dimensional vector space $V(F)$ then $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$

(14) Let $U(F)$ and $V(F)$ be two vector spaces and $S = \{\alpha_1, \alpha_2 \cdots \alpha_n\}$ be a basis of U. Let $\{\delta_1, \delta_2 \cdots \delta_n\}$ be a set of 'n' vectors in V. then there exists a unique linear transformation $T: U \to V \ni T(\alpha_i) = \delta_i$

(15) Let $T: U(F) \to V(F)$ be a L.T then the Range space $R(T)$, & the null space $N(T)$ are subspaces of $V(F)$.

[Prove $a\alpha + b\beta \in R(T)$ & $N(T)$].

(16) show that a linear transformation $T: U \to V$ is non-singular $\Leftrightarrow T$ is 1-1

(17) state & Prove Rank-nullity thm.

(18) state & prove Fundamental thm of homomorphism.

(19) Find the nullspace, range, rank and nullity of the transformation $T: R^2 \to R^3$ defined by $T(x, y) = (x+y, x-y, y)$

(20) show that the linear operator T on R^3 is invertible and find T^{-1}. $T(x, y, z) = (2x, 4x-y, 2x+3y-z)$

(Show that T is non-singular and find T^{-1}).

UNIT - II

MATRICES & INNER PRODUCT SPACES

(1) Find the characteristic roots and vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (Practice the Problem)

(2) State & Prove CALEY-HAMILTON thm.

(3) Show that the matrix $A = \begin{bmatrix} 5 & 6 & 6 \\ 0 & 7 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ is diagonalizable and find the diagonal matrix.

(4) State & Prove Sylvester's law of nullity.

(5) The characteristic vectors corresponding to distinct characteristic roots of a matrix are LI

(6) If the characteristic roots of a square matrix of A are $\lambda_1, \lambda_2 \cdots \lambda_n$ then prove that the characteristic roots of A^2 are $\lambda_1^2, \lambda_2^2 \cdots \lambda_n^2$.

(7) If $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ verify Cayley-Hamilton then the find A^{-1}.

(8) Determine the modal matrix P of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. verify and $P^{-1}AP$ is a diagonal matrix. (Practice the Problem)

(9)(a) Define Inner product space

(b) Let $V(C)$ be the vector space of all continous complex valued functions on the interval $[0,1]$. for $f, g \in V$ if $\langle f, g \rangle = \int_0^1 f(t)\overline{g(t)}\,dt$ then V is an inner Product space.

(10) State and Prove Cauchy's Inequality thm.

(11) State and Prove Triangle Inequality.

(12) The vectors α, β of a real inner Product space $V(F)$ are orthogonal $\Leftrightarrow \|\alpha+\beta\|^2 = \|\alpha\|^2 + \|\beta\|^2$

(13) In an inner product space any orthonormal set of vectors is LI.

(14) Two vectors α, β in an unitary space $V(C)$ are such that $\langle \alpha, \beta \rangle = 0 \iff \|a\alpha + b\beta\|^2 = |a|^2 \|\alpha\|^2 + |b|^2 \|\beta\|^2 \ \forall a, b \in C$.

(15) State & prove Parallelogram law

(16) Every finite dimensional inner product space has orthonormal Basis (Gram-schmidt thm)

(17) $S = \{(2,1,3)\,(1,2,3)\,(1,1,1)\}$ is a basis of R^3 construct an orthonormal basis by using G-S orthogonalisation Process.

(18) State & Prove Bessel's inequality

(19) State & Prove Parseval's Identity.

(20) If α, β are two vectors in an IPS, then α, β are LD $\iff |\langle \alpha, \beta \rangle| = \|\alpha\| \|\beta\|$.

UNIT - III

MULTIPLE INTEGRALS

(1) Prove that the sufficient condition for the existance of the line integral.

(2) The necessary and sufficient condition for the integrability of a bounded function f over a rectangle R is that to $\varepsilon > 0$. $\exists$ $\delta > 0$ such that $(P, f) < \varepsilon$ for every Partion P of R whose norm is less than (or) equal to δ.

(3) (i) If $f(x,y)$ is integrable on $R = [a, b : c, d]$
(ii) $h(y) = \int_a^b f(x,y)\,dx$ exists for each $y \in [c, d]$ then $h(y)$

is integrable on $[c,d]$ and $\int_c^d \left(\int_a^b f(x,y)\,dx\right) dy = \iint_E f(x,y)\,dx\,dy$

(4) sketch the region of integration for the integral $\int_{x=0}^{x=2} \int_{y=x^2}^{y=2x} (4x+2)\,dy\,dx$ and write an equivalent integral with the order of integration reversed.

(5) By changing the order of integration and hence evaluate $\int_0^1 dy \int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} \frac{dx}{(x^2-2x+y-3)^2}$

(6) change the order of integration and hence show that $\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}} = \frac{\pi}{2} \log\left(\frac{2e}{1+e}\right)$.

(7) Evaluate $\iint e^{x^2+y^2}\,dy\,dx$, where E is the semi-circular region bounded by the x-axis, and the curve $y=\sqrt{1-x^2}$.

(8) Evaluate $\iint_E \frac{xy\,(x+y)^2}{x^2+y^2}\,dx\,dy$, where E is the region bd by $y=0$, $y=x$, $x^2+y^2=a^2$ in the first quadrant.

(9) Find the value of $\int (x+y^2)\,dx + (x^2-y)\,dy$ taken in the clockwise sense along the closed curve C formed by $y^2=x$ and $y=x$ between $(0,0)$ and $(1,1)$

(10) Evaluate $\iint xy\,(x^2+y^2)\,dx\,dy$ over $[0,1;0,1]$.

(11) show that $\phi(h) = \int_h^1 \left(\int_h^1 \frac{x-y}{(x+y)^3}\,dy\right) dx$ is not continuous for $h=0$.

(12) state & Prove Fubini's thm.

(13). Evaluate $\iint_E \sqrt{4x^2-y^2}\, dx\, dy$ where E is the triangle

bounded by the lines $y=0$, $y=x$, $x=1$.

(14) Evaluate $\iint_E xy(x+y)\, dx\, dy$ where E is the region bounded by $y=x^2$, $y=x$.

(15) Evaluate $\iint_E xy\, dx\, dy$, where $E = \{xy=1,\ y=0,\ y=x,\ x=2\}$.

(16) Evaluate $\iint_E \frac{y^2}{1+x^2}\, dx\, dy$ on $[-1,1 : 0,2]$.

(17) Find the area of the cap cut from the hemi-sphere $x^2+y^2+z^2=2$, $z\geq 0$ by the cylinder $x^2+y^2=1$. (area of surface)

(18) Find the area of the surface cut from the cylinder $x^2+z^2=4$ by the cylinder $x^2+y^2=a^2$.

(19) Find the length of the cardiod given by

$r = a(1-\cos\theta)$

(20). ~~Find the~~ using polar coordinates,

show that $\int_0^1 dx \int_0^x \sqrt{x^2+y^2}\, dy = \frac{1}{6}\left[\sqrt{2}+\log(1+\sqrt{2})\right]$

UNIT - IV

VECTOR CALCULUS

(1) If $\bar{r} = a\cos t\, i + a\sin t\, j + at\tan\alpha\, k$ find $\left|\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2}\right|$

and $\left|\frac{d\bar{r}}{dt}\ \frac{d^2\bar{r}}{dt^2}\ \frac{d^3\bar{r}}{dt^3}\right|$

(2) If $a = x+y+z$, $b = x^2+y^2+z^2$, $c = xy+yz+zx$, Prove

$[\text{grad } a,\ \text{grad } b\ \ \text{grad } c]$ (or) $[\nabla a\ \ \nabla b\ \ \nabla c] = 0$.

(3) If a is a constant vector, Prove that curl $\frac{a \times \bar{r}}{r^3} = \frac{-a}{r^3} + \frac{3\bar{r}}{r^5}(a \cdot \bar{r})$

(4) show that (i) $\nabla r = \frac{\bar{r}}{r}$, (ii) $\nabla\left(\frac{1}{r}\right) = \frac{-\bar{r}}{r^3}$ (iii) $\nabla r^3 = 3r\bar{r}$

(5) If $f = xy^2 i + 2x^2yz j - 3yz^2 k$ then find div f, curl f at $(1,-1,1)$.

$$\text{grad } \phi = \nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z} \quad (\phi \text{ scalar})$$

$$\text{div } f = \nabla \cdot f = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} \quad (f \text{ vector})$$

$$\text{curl } f = \nabla \times f = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix} \quad (f = f_1 i + f_2 j + f_3 k)$$

(6) P.T div (curl f) $= 0$ & curl (grad ϕ) $= 0$.

(7) show that (i) $\nabla^2(r^n) = n(n+1)r^{n-2}$ (ii) $\nabla^2\left(\frac{1}{r}\right) = 0$.

(8) Prove that (i) $\nabla \cdot (\phi A) = (\nabla\phi) A + \phi(\nabla \cdot A)$.

(ii) $\nabla \times (\phi A) = (\nabla\phi) \times A + \phi(\nabla \times A)$.

(9) Prove that curl $(A \times B) = A \text{ div } B - B \text{ div } A + (B \cdot \nabla)A - (A \cdot \nabla)B$.

(10) show that div $(A \times B) = B \cdot \text{curl } A - A \cdot \text{curl } B$.

(11) Find $\int_c F \cdot dr$ where $F = xyi + yzj + zxk$ and the curve c is $r = ti + t^2 j + t^3 k$, t varying from -1 to 1.

(12) Find the work done in moving a Particle force

field $F = 3x^2 i + (2xz - y) j + z k$ along the straight line from $(0,0,0)$ to $(2,1,3)$ then find $\int_c F \cdot dr$

(13) If $F = 4xz\, i - y^2 j + yz\, k$. evaluate $\int_S F \cdot N\, ds$ wher S is the surface of the cube bounded by $x=0,\ x=a,\ y=0,\ y=a,\ z=0,\ z=a$.

(14) Evaluate $\int_S F\, N\, ds$ where $F = zi + xj - 3y^2 z\, k$ and S is the surface $x^2+y^2=16$ included in the first octant between $z=0$ and $z=5$

(15) state & Prove (i) Gauss divergence thm

(ii) Green's thm

(iii) stokes thm.

(16) If $F = 2xz\, i - xj + y^2 k$ evaluate $\int_V F\, dv$ where V is the region bounded by the surfaces $x=0,\ x=2,\ y=0,\ y=6,\ z=x^2,\ z=4$.

(17) use Gauss thm to evaluate $\int_S (x^3 - yz) i - 2x^2 y j + zk) \cdot N ds$ over the surface of cube bounded by the coordinate planes $x=y=z=a$

(18) If $F = 3xy\, i - y^2 j + yz\, k$. the verify Gauss divergence thm wher S is the surface of the cube bounded $x=0,\ x=a,\ y=0,\ y=a,\ z=0,\ z=a$.

(19) Evaluate $\int_c (\cos x \sin y - xy)\, dx + \sin x \cos y \cdot dy$. by Green's thm where c is the circle $x^2+y^2=1$.

(20) verify Green's thm in the plane for $\oint_c (xy+y^2)\,dx + x^2 dy$, where c is the closed curve of the region bounded by $y=x$ and $y=x^2$.

(21) verify stokes thm for $F=-y^3 i + x^3 j$, where S is the circular disc $x^2+y^2 \le 1$, $z=0$.

(22) verify stokes thm for $A=(2x-y)i - yz^2 j - y^2 z k$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and c is its boundary.

(23) verify Green's thm in the plane $\oint_c (3x^2-8y^2)\,dx + (4y-6xy)\,dy$

$y=\sqrt{x}$ and $y=x^2$

(24) Find $\iint_S (ax^2+by^2+cz^2)\,ds$ over the sphere $x^2+y^2+z^2=1$.

(use gauss thm)

(25) show that $\iint_S (axi+byj+czk)\cdot N\,ds = \frac{4\pi}{3}(a+b+c)$

All the Best