

# UNIT-1 Important Questions

- 1) if  $H, K$  are subspace of  $V(F)$  then p.f  $H \cap K$  is subspace of  $V(F)$  and  $H \cup K$  is not subspace of  $V(F)$
- 2) let  $v_1, v_2, \dots, v_p$  are vectors in  $V(F)$  then p.f  $\text{span}\{v_1, v_2, \dots, v_p\}$  is subspace of  $V(F)$
- 3) if  $u_1, u_2, \dots, u_p, v_1, v_2, \dots, v_p$  are vectors in  $V(F)$  and  $H = \text{span}\{u_1, u_2, \dots, u_p\}$ ,  $K = \text{span}\{v_1, v_2, \dots, v_p\}$  then p.f  $H + K = \text{span}\{u_1, u_2, \dots, u_p, v_1, v_2, \dots, v_p\}$
- 4) if  $T: V \rightarrow W$  is a linear transformation then p.f
  - a)  $\text{ker}(T)$  is subspace of  $W$
  - b)  $\text{Range}(T)$  is subspace of  $W$ .
- 5) Define lin. dep & lin. Independent and state and prove spanning set theorem.
- 6) Define Basis. state and prove unique Representation theorem.
- 7) let  $B = \{b_1, b_2, \dots, b_n\}$  is a basis set of a vector space  $V(F)$ . Then p.f any set which is containing more than  $n$ -vectors is lin. Independent.
- 8) state and prove Basis theorem.
- 9) Define Null space and column space. A null space of order  $m \times n$  matrix of 'A' is subspace of  $R^n$ .

# UNIT - 1

1) Define Rank of a matrix - state and prove Rank Theorem.

2) Determine Basis of Row space, column space, nullspace following matrices. And verify Rank Theorem

a)  $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$       b)  $A = \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3) P.P Eigen values of a triangular matrix is same as its diagonal element.

4) If  $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$  then find the basis of the eigen space corresponding  $\lambda = 1, 2, 3$

5) find Eigen values and eigen vectors of  $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$

6) If  $v_1, v_2, \dots, v_r$  are Eigen vectors of corresponding distinct Eigen values  $\lambda_1, \lambda_2, \dots, \lambda_r$  of  $n \times n$  matrix of "A" then the set  $\{v_1, v_2, \dots, v_r\}$  is lin. Independent.

7) If  $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$  then find Eigen vectors corresponding Eigen values  $\lambda = 3$

## UNIT - III

① if "A" and "B" are two similar matrices. Then P.P they have the same char. poly and hence has the same Eigen values.

② state and prove Diagonalisation theorem

3) Test the Diagonalisability of  $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

4)  $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$  Test the diagonalisability then find  $A^e$ .

5) if  $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$   $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$  then compute  $A^2, A^4$

if  $A = PDP^{-1}$

## UNIT - IV

1) state and prove pythagorean theorem

2) state and prove parallelogram theorem

3) if  $S = \{u_1, u_2, \dots, u_n\}$  is a orthogonal set of vector in  $R^n$ . then P.P "S" is Lin. Pnd. where "S" has non zero vectors

4) If  $\{u_1, u_2, \dots, u_n\}$  is a orthogonal basis for a subspace "W" of  $R^n$ . for each "y" in "W" in the weight on the linear combination  $y = c_1 u_1 + c_2 u_2 + \dots + c_n u_n$  is Given by

$$c_i = \frac{y \cdot u_i}{u_i \cdot u_i} \quad (i = 1, 2, \dots, n)$$

5) If  $U$  is a  $m \times n$  matrix of  $m \times n$  if  $U$  has orthogonal columns  $\Leftrightarrow U^T U = I_n$

6) if  $u_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$ ,  $u_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ ,  $u_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$  and  $x = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$

then show that  $\{u_1, u_2, u_3\}$  is orthogonal basis of  $\mathbb{R}^3$ . Express  $x$  as lin. comb. of  $u_1, u_2, u_3$ .

7)  $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \right\}$  and  $x = \begin{bmatrix} 8 \\ -4 \\ -3 \end{bmatrix}$  then  $S$  is orthogonal basis of  $\mathbb{R}^3$ . Express  $x$  as lin. comb. of  $u_1, u_2, \dots, u_n$ .

8) Find the projection of  $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$  and  $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ . also write  $y$  as sum of two orthogonal vectors one in span  $\{u\}$  and orthogonal to  $u$ .

9) problems on projection

a)  $y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ ,  $u = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

b)  $y = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ ,  $u = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

*(Faint background notes and calculations)*

Let us consider  $u = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$  and  $v = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$  are orthogonal.

$\|u\| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\|v\| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$

Projection of  $y$  on  $u$  is  $\frac{y \cdot u}{\|u\|^2} u = \frac{1(-1) + (-1)(-1)}{2} u = \frac{-1 + 1}{2} u = 0$

Projection of  $y$  on  $v$  is  $\frac{y \cdot v}{\|v\|^2} v = \frac{1(-1) + (-1)(3)}{10} v = \frac{-1 - 3}{10} v = \frac{-4}{10} v = \frac{-2}{5} v = \frac{-2}{5} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2/5 \\ -6/5 \end{bmatrix}$

Projection of  $y$  on  $\text{span}\{u, v\}$  is  $0 + \begin{bmatrix} 2/5 \\ -6/5 \end{bmatrix} = \begin{bmatrix} 2/5 \\ -6/5 \end{bmatrix}$

Orthogonal component is  $y - \begin{bmatrix} 2/5 \\ -6/5 \end{bmatrix} = \begin{bmatrix} 7 - 2/5 \\ 6 - (-6/5) \end{bmatrix} = \begin{bmatrix} 33/5 \\ 36/5 \end{bmatrix}$

$y = \begin{bmatrix} 2/5 \\ -6/5 \end{bmatrix} + \begin{bmatrix} 33/5 \\ 36/5 \end{bmatrix}$